Segmentation of Magnetic Resonance Brain Imaging Based on Graph Theory

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**Abstract**

Segmentation of brain images especially from magnetic resonance imaging (MRI) is an essential requirement in medical imaging since the tissues, edges, and boundaries between them are ambiguous and difficult to detect, due to the proximity of the brightness levels of the images.

Material and Methods: In this paper, the graph-based method is proposed to solve the segmentation of MRI brain images wherein a weighted undirected graph is assigned to the image with each edge of the graph corresponding to an image pixel. The edge weight between two nodes demonstrated the similarity between two pixels of the image. Thereafter, a cost function, such as relative extremes and turning point, was assigned to the graph, which matched the derivation of the function. Minimization of this cost function, which was equivalent to the shortest path in a graph, led to image segmentation.

Results: The advantage of the graph method over other methods is the simultaneous construction of spatial information at a high rate. Moreover, this method is implemented on the pixels in the space and can partition MRI brain images with low error in an effort to improve the previous methods. The comparisons demonstrated that the accuracy of the MRI brain image segmentation would be improved through the application of the present method.

Conclusion: The obtained results of the current study indicated the high accuracy of the proposed method (about 97.5%), compared to other similar methods. Therefore, this method can accurately distinguish various types of brain MRI tissues and have clinical applications.

**Introduction**

Image segmentation is defined as dividing an image into some sections or classes which correspond to various objects. This task can be considered an essential procedure in medical image processing since some problems, such as noise and blurring, are encountered in the boundaries resulting in the reduction of information in the extracted image \([1,2]\).

Segmentation aims to partition image to distinct homogeneous areas or find the boundaries between these areas. It is expected that these regions (or equivalently, the obtained boundaries) be coincident with actual objects or a portion of them, and the resultant areas ideally match those emerged from human brain segmentation. Segmentation plays a vital role in many image processing and machine vision applications since this task influences the outcome of the entire analysis \([3-5]\).

A good image segmentation method for grey images should have the following features:

1. The segmented areas should be uniform and homogeneous regarding one or more features, such as color and texture.
2. Each area should be simple and without large holes.
3. The adjacent areas should be significantly different with respect to the same homogeneity feature. Moreover, the location of the borders of each area should be accurate.

The principal image segmentation methods can be divided as \([6]\):

1. Feature-based segmentation, including clustering, K-means adaptive segmentation, and image histogram thresholding techniques
2. Image-based segmentation, including splitting and merging, region growing, graph-based methods, edge detection-based methods, and methods based on neural networks
3. Physics-based methods

A summary of the advantages and disadvantages of each method is provided in Table 1. In this regard, the segmentation of brain MRI is an important task in many medical applications since the final analytical outcome and the other different processing steps are completely affected by the precise segmentation of

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anatomical regions. Surgical planning, image-guided interventions, analysis of brain development, and delineation of lesions are among the various applications of brain MRI segmentation [3]. Furthermore, this task can be performed to analyze different brain tissues [4].

Many studies have addressed the classification methods used in MRI images, such as binary morphology operation, K-means, watershed, Markov’s random field’s, area growth, artificial neural networks, and deformable models methods. The specifications of the aforementioned methods are as follows:

Table 1. Comparison of different image segmentation methods

<table>
<thead>
<tr>
<th>Image segmentation methods</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tbody>
<tr>
<td>Feature-based image segmentation methods</td>
<td></td>
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</tr>
<tr>
<td>Clustering</td>
<td>Generally unsupervised methods</td>
<td>Pixel location information is not used. Number of clusters is needed in some methods. Adaptation of resulted clusters to image areas is difficult.</td>
</tr>
<tr>
<td>Adaptive methods</td>
<td>Adaptation to various image areas as an important property Spatial continuity in the resultant local areas</td>
<td>Local maximums can defect the posterior probability maximization process</td>
</tr>
<tr>
<td>Image histogram thresholding</td>
<td>No need for initial information regarding the image Presence of simple and fast algorithms</td>
<td>Spatial information is neglected. Thresholding on multi-dimensional space is difficult. The peaks due to noise result in ambiguities in segmentation.</td>
</tr>
<tr>
<td>Splitting and merging methods</td>
<td>Use of pixel location information Provision of acceptable results on images with homogeneous regions</td>
<td>It is difficult to define a color homogeneous criterion. Quaternary tree structure has many deficiencies.</td>
</tr>
<tr>
<td>Graph-based methods</td>
<td>Simultaneous inclusion of space location and features in the created graph Presence of high-speed greedy algorithms for some methods</td>
<td>Some methods are computationally intensive with very low speed.</td>
</tr>
<tr>
<td>Edge detection-based methods</td>
<td>Better edge detection and enhanced results by color information Good results in objects detection by active contours</td>
<td>Definition of gradient functions is difficult for colored images. Noise or low-contrast images weaken the performance of edge detection methods and can results in confusion.</td>
</tr>
<tr>
<td>Neural network-based methods</td>
<td>Fast computations due to the possibility of parallel execution of algorithms High suitability for some specific applications, such as medical images and face recognition</td>
<td>Color information leads to network complexity. The number of categories should be specified in the training phase</td>
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**Binary morphology operation method**

This algorithm is based on mathematical morphology with the basic idea of using Multi-scale open reconstruction of mathematical morphology to transform the image continuously. Thereafter, the decision about the termination is made based on the results of the adjacent image transformation. The final segmentation results are then obtained by further processing of images after termination. When this method is applied to images with simpler tissues, it usually produces suitable results. However, there is a weak continuity in some regions between the brain tissues and other tissues, like eye regions, despite the similarity of the grey areas in these regions. In this regard, the incorporation of the binary morphology operation method can strengthen this weak continuity using binary images before the operation and in so doing makes the subsequent separation process difficult. Therefore, the main challenge of binary morphology operation method is the selection of a suitable threshold level to create a two-level image [7-10].

**K-mean method**

This algorithm is an iterative technique used to divide an image into K clusters in which K cluster centers are either selected randomly or based on some heuristic method. Each pixel is then assigned to one cluster by the calculation of the distance between the pixels and the cluster centers and minimization of this distance. Thereafter, the centers of the cluster are recomputed through averaging all of the pixels in the cluster. This algorithm is guaranteed to converge; however, it may not return the optimal solution. The quality of the solution depends on the initial set of clusters and the value of K. Moreover, this method ignores the spatial correlations of neighboring points in the calculations. Additionally, since the final result
depends on the initially selected categories, it does not give the same result if repeated.

The K-means method results in the creation of categories with a minimum internal variance; however, this is a local and not necessarily a global minimum. From a statistical point of view, if the categories are created by normal distributions with different means and similar variances, finding the resultant categories by the K-means method is equivalent to maximum likely estimation of the center of categories. This is the main drawback of the K-means method since the optimal result is only achieved when the regions have a normal distribution with the same variance [11-13].

Watershed method
The image gradient magnitude is considered a topographic surface in this method. The pixels with the highest gradient magnitude intensities corresponding to watershed lines represent the region boundaries. Water placed on any pixel enclosed by a common watershed line flows downhill to a common local intensity minimum. Pixels draining to a common minimum form a catchment basin which represents a segment. One defect of this method is over-segmentation wherein the image is divided into many undesirable regions. Therefore, it is necessary to perform a post-processing phase to combine these regions for obtaining the desired regions. Furthermore, long execution time is another disadvantage of this method [14-16].

Markov’s random field’s method
The strong mathematical foundation of Markov’s random field (MRF) and its ability to provide a global optimum proved this method as the foundation for new research in such domain as image analysis, denoising and segmentation. The MRF is completely described by its probability distribution, marginal probability distribution, smoothing constraint, and criterion for updating value. Some of the disadvantages of this method include the conceptual complexity of the algorithm structure, requirement for powerful computational algorithms, and long execution time [17,18].

Area growth algorithm
In this method, the neighboring pixels in one region are assumed to have similar values. Regarding this, the main procedure in this algorithm is the comparison of each pixel with its neighbors. If the defined similarity criterion is satisfied, the pixel belongs to the same cluster as one or more of its neighbors. In this respect, the definition of the similarity criterion is considered one of the important steps of this method affecting the results. The main drawback of this method is the selection of the points of origin and homogeneous criteria problem [19-21].

Neural networks methods
This method is one of the trainable segmentation algorithms constructed to process small areas in an image through one or a set of artificial neural network. The decision-making mechanism characterizes the areas of an image according to the category recognized by the neural network. The main drawback of these methods is the need for a network training phase and pre-specification of regions. Considering the above survey, the conventional image segmentation methods can be concluded to fall short of giving suitable results for the brain tissue segmentation using MRI images. This can be due to the complexity of these images and large variations in grey areas between the regions. Regarding this, the application of hybrid methods can yield more desirable results for the segmentation of brain tissues from MRI image. Therefore, due to the accuracy and automaticity of the graph-based method, it was adopted in this paper to achieve the desired goal of the fast segmentation of MRI brain tissue images [22-24].

The graph-based segmentation method initiates by the automated detection of one or several points demonstrating the ‘object’. In the proposed technique, a weighted undirected graph is assigned to the image by considering a graph edge for all its constituent pixels. More similarity between pixels leads to high weight between nodes. The assignment of cost function to the graph and its minimization lead to image segmentation. The superiority of the graph method over other aforementioned methods is the simultaneous and appropriate construction of spatial information in the built graph.

Moreover, there are fast algorithms for some methods. There are several graph-based methods for the cost function minimization and MRI brain image segmentation, such as normalized cuts [25], minimum cut [26], partitioning using isoperimetric [27], random walker [28], and segmentation based on minimum spanning tree [29]. These popular graph-based image segmentation methods have various applications in biomedical imaging [30, 31].

In the graph-based approach of this paper, the cuts are opened and converted to the polar coordinates. A graph is then assigned to an opened form, and an arbitrary path is selected in this graph. The next step of the proposed approach is considering the energy function as the sum of path cost and smoothness term to the arbitrary path. To minimize the cost function, a recursive form of dynamic programming (DP) is utilized, and optimization problem leads to the attainment of the shortest path and image segmentation. This algorithm is completely different from the other graph-based segmentation methods in terms of the cost function and optimization methodology.
Materials and Methods

Materials
This paper employed the BRATS 2015 MRI dataset. This dataset includes several images provided by BRATS 2012, 2013, as well as the National Institutes of Health Cancer Imaging Archive [31].

Research procedures
The objective of this paper was the segmentation of MRI brain images. In this regard, to solve the segmentation problem, the input image was converted to a graph after the implementation of initial processing procedures. The graph was then applied to the image, and the image was segmented. The graph-based image segmentation methods dealing with a graph were defined with a function named \( G \). This function was described as \( G(N,E) \), where \( N \) was a set of nodes, and \( E \) is a set of weighted edges. Each node represents a pixel in the image, while each edge connects the neighboring pixels. The assigned weight to each edge is based on some features of the starting and ending pixels of the edge. In the first graph-based method, a threshold value and local measurement were used for segmentation computations. There are two image segmentation methods based on graph formulation:

1) Area-based method: In this method, each node in the graph represents a set of interconnected image pixels.
2) Pixel-based method: In this method, each node in the graph represents one pixel of the image. This method does not necessarily have satisfactory performance for complex images with distinct objects.

Image segmentation using graph method
First, it was assumed that the statistical graph method applied spatially on the pixels can segment the MRI brain images with low percentage error and in doing so enhance the previous methods. This technique can be implemented as follows:

1) The cuts are opened and converted to the polar coordinates.

Each polar image includes a vector \( (r, \theta) \) and an angle \( \theta \), and can be represented as \( (r, \theta) \). The positive rotation is supposed to be in a counter-clockwise direction. The black part, determined by the black line, is called background, while the white part in the periphery of the skull is represented as a continuous line called borderline. Figure 1 shows a brain segmentation sample, Figure 2 shows an image in polar coordinate, and Figure 3 is obtained with the rotation of Figure 2 with an angle of \( \theta \), which is the opened form of Figure 2. The relationship between polar and Cartesian coordinates is demonstrated in Figure 4 and Equations 1-4.

\[
x = r \cos \theta \\
y = r \sin \theta \\
r = \sqrt{x^2 + y^2} \\
\theta = \tan^{-1} \frac{y}{x}
\]

However, two problems arise in the application of this technique. Firstly, the borderline should be in the same location for all the columns of the whole image, described by function \( F(x) \).
Figure 4. Relationship between polar and Cartesian coordinates in image segmentation

When rotating with an angle of $\theta$, the rotation is on the first vector by selecting some points. The number of points should neither be too high nor too small since a high number of points does not show the details and a low number does not produce an acceptable image. The procedure is continued until the achievement of the first white point. The same procedure is performed on the second vector until the achievement of the first white point. The accumulation of the white points of each vector results in the borderline formation.

This procedure can be performed by assigning a graph to the opened polar form of the image (see Figure 3). In this graph, the cost value of pixels in the image is the number of nodes of the graph, where the graph connections show the limitations to find the borderline.

The $(x_1, x_2, \ldots, x_n)$ demonstrated in Equation 5 is an arbitrary path in this graph, and the energy of this path is defined as:

$$E = E(x_1, x_2, \ldots, x_n)$$

(5)

Minimization of the energy function $E = E(x_1^*, x_2^*, \ldots, x_n^*)$ leads to finding the optimal path $(x_1^*, x_2^*, \ldots, x_n^*)$. The energy function can be described in detail as follows:

$$E(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} C(x_i) + \sum_{i=2}^{n} d(x_{i-1}, x_i)$$

(6)

In Equation 6, $C(x_i)$ is the cost of the path connected to $x_i$, while the smoothness term $d(x_{i-1}, x_i)$ is the cost value between two neighboring paths ($x_{i-1}$ and $x_i$). Although $d(x_{i-1}, x_i)$ is a geometrical cost obtained based on the path features, $C(x_i)$ could be calculated based on the image intensity data. The DP as one of the numerical techniques vastly applied in optimization problems is utilized to solve the minimization problem [32].

To implement DP, the path energy function is divided into some sub-functions. Defining this function for the first and second states as $E(x_1) = c(x_1)$ and $E(x_1, x_2) = E(x_1) + c(x_2) + d(x_1, x_2)$, the energy function is converted to:

$$E(x_1, x_2, \ldots, x_n) = E(x_1, x_2, \ldots, x_{n-1}) + c(x_n) + d(x_{n-1}, x_n)$$

(7)

The DP minimizes the energy function using the following recursive equation:

$$C_i(x_i) = c(x_i) + \min_{x_{i+1}} (C_{i+1}(x_{i+1}) + d(x_{i+1}, x_i))$$

(8)

Where the minimum aggregate cost of the shortest path between each state and the beginning of the graph is saved in $C_i$. Afterward, the beginning point of the global shortest path of the graph is obtained calculating the minimum of all cost function $C_n$, as:

$$x_n^* = \arg \min_{x_n} C_n(x_n)$$

(9)

Where the aggregate cost function is minimum.

$$C_n(x_n^*) = E(x_1^*, x_2^*, \ldots, x_n^*) = \min(E)$$

(10)

Finally, the global shortest path is obtained as follows:

$$x_i^* = \arg \min_{x_i} (C_i(x_i) + d(x_i, x_i^*))$$

(11)

Figure 5. Appropriate derivation of image boundary based on the selected cost function.
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Figure 6. Representation of the minimum

Figure 7. Employing the Dynamic Programming to find the image path

Figure 8. Selection of the minimum value in each column based on the image path that is modeled applying a graph

Figure 9. Obtaining the minimum path

Figure 10. Obtaining the minimum path and applying it on the image

Figure 11. Conversion of the original image to a polar image
The proposed segmentation method based on graph theory is now described. In this graph, the cost value of pixels in the image (Figure 5) is the number of nodes of the graph, where the graph connections show the limitations to find the borderline. The minimization of the limitations leads to searching for the minimum part of the graph, and the minimum part is found at the end. The procedure is more clearly described in figures 6-11.

**Results**

The performance of the proposed method was evaluated in two stages. Figure 12 illustrates the implementation of all stages of the proposed method on an MRI brain image. The original MRI image is the input of the algorithm, and the segmented image is the output. All of the simulation results were obtained by the implementation of the proposed method using the MATLAB 2012a software. As illustrated, the proposed approach had a good performance in finding the boundaries between various regions of the brain and skull image segmentation.

In the second phase of the simulation, the effectiveness of the proposed method in the detection of brain tumour was assessed and compared to other methods. The selected algorithm for this comparison was the active contours driven by local Gaussian distribution fitting energy segmentation method. Figure 13 depicts the implementation of the proposed segmentation and active contours methods on two MRI brain images of the introduced dataset. As indicated, the proposed method had better accuracy in brain tumor detection, compared to the active contours method. In addition, the quantitative evaluation of the segmentation accuracy of these methods for the brain MRI images presented in Figure 13 is shown in Table 2. The mentioned values in this table included the accuracy percentages calculated according to the Dice coefficient, defined as (12):

\[
\text{Dice}(P,T) = \frac{2|P \cap T|}{|P| + |T|}
\]

Where \(P\) and \(T\) denote the set of pixels labeled as the “ground truth” and MRI segmentation, respectively. Additionally, \(|P|\) and \(|T|\) signify the number of elements in \(P\) and \(T\). The Dice coefficient is within the range of \(0 \leq \text{Dice}(P,T) \leq 1\), and has a value of 0 if there is no overlap between the two segmentations and 1 if both segmentations are identical. The Dice score normalizes the number of true positives to the average size of the two segmented areas [31]. The results presented in Table 2 confirm the improvement in the segmentation accuracy of the proposed method, compared to the active contours.

![Skull image segmentation and extraction of the brain tissue from the skull](image-url)
The MRI brain tissues are of great interest for segmentation using different algorithms. According to the available literature, the graph-based algorithm is a new approach with a potential for the segmentation of various types of medical images. This method successfully facilitates the combination of spatial information with pixel location information. Moreover, the optimal result is global, and the information of the whole image is used in segmentation. Another desirable feature of graph-based methods is related to two intuitive clustering concepts wherein many similarities can be found inside the segments and low similarity between different segments. Both of these concepts are simultaneously satisfied in this method. The framework of this method is assigning a weighted undirected graph to the image and segmenting this graph according to a predefined criterion. Each node of the graph corresponds to a pixel of the image, and the weight of the graph edge represents the similarity between the corresponding image pixels.

Table 2. Comparison of segmentation accuracy of the active contours driven by local Gaussian distribution fitting energy method and the proposed method

<table>
<thead>
<tr>
<th></th>
<th>Active contours</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 13. (a)</td>
<td>89.23</td>
<td>97.45</td>
</tr>
<tr>
<td>Figure 13. (d)</td>
<td>82.34</td>
<td>94.61</td>
</tr>
</tbody>
</table>

Discussion

The MRI brain tissues are of great interest for segmentation using different algorithms. According to the available literature, the graph-based algorithm is a new approach with a potential for the segmentation of various types of medical images. This method successfully facilitates the combination of spatial information with pixel location information. Moreover, the optimal result is global, and the information of the whole image is used in segmentation. Another desirable feature of graph-based methods is related to two intuitive clustering concepts wherein many similarities can be found inside the segments and low similarity between different segments. Both of these concepts are simultaneously satisfied in this method. The framework of this method is assigning a weighted undirected graph to the image and segmenting this graph according to a predefined criterion. Each node of the graph corresponds to a pixel of the image, and the weight of the graph edge represents the similarity between the corresponding image pixels.

Conclusion

The MRI brain tissues are of great interest for segmentation using different algorithms. According to the available literature, the graph-based algorithm is a new approach with a potential for the segmentation of various types of medical images. This method successfully facilitates the combination of spatial information with pixel location information. Moreover, the optimal result is global, and the information of the whole image is used in segmentation. Another desirable feature of graph-based methods is related to two intuitive clustering concepts wherein many similarities can be found inside the segments and low similarity between different segments. Both of these concepts are simultaneously satisfied in this method. The framework of this method is assigning a weighted undirected graph to the image and segmenting this graph according to a predefined criterion. Each node of the graph corresponds to a pixel of the image, and the weight of the graph edge represents the similarity between the corresponding image pixels.
method, and segmentation of this graph according to a predefined criterion. Qualitative evaluations of several images utilizing MRI datasets validated the superior performance of the proposed approach in the detection of tumor and various regions of the brain tissues, compared to the active contours driven by local Gaussian distribution fitting energy.

References


