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Analysis of Accuracy of Interpolation Methods in Estimating the Output Factors for Square Fields in Medical Linear Accelerator

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ABSTRACT

Introduction: To estimate the accuracy levels of Lagrange, Newton backward interpolation, and linear interpolation methods in estimating the output factors for square fields used in linear accelerator for 6 MV photons at various depths.

Materials and Methods: Ionization measurements were carried out in radiation field analyzer in linear accelerator for 6 MV beams at the depths of 5 and 10 cm by 0.6 cc Farmer-type ionisation chamber. Dosimetry was performed by ion collection method with 0.5 cm^2 interval for square fields from 4 × 4 cm^2 to 40 × 40 cm^2 field sizes. The measured output factor values for 10 square field sizes with equal interval were taken for interpolating the intermediate square field size values. The Lagrange and Newton backward methods were used for predicting the intermediate output factors.

Results: The percentage of deviation from the measured value was estimated for all the three methods. The calculated output factor values of the two proposed methods were compared with the standard linear interpolation method used in routine clinical practice. It was observed that the Lagrange and Newton backward methods were not significantly different from the measured value (P=0.77). The linear interpolation values were significantly different from the measured value (P<0.01).

Conclusion: It is recommended to use the Lagrange and Newton backward interpolation methods to estimate the intermediate output factors to increase accuracy in treatment delivery. The routine linear interpolation method can be applied only for small intervals. This proposed interpolation method is highly associated with the measured values in all the interval levels.

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Introduction

The success of radiotherapy treatment depends on the accurate delivery of prescribed dose, which was determined by the measurements under non-reference conditions [1]. The dose can be calculated knowing the output at reference depth, tissue phantom ratios or the tissue maximum ratios, and the relative output factors [2]. The accuracy of the delivered dose must be lower than ±5% of the prescribed dose [3-5]. Output factor is one of the essential measurements required for accurate dose calculation at any point. The in-water output ratio, S_{cp}, for a field size is defined as the ratio of the absorbed dose for the used collimator setting to the absorbed dose for the reference field size at the same depth of reference [6]. The output factor increases with the collimator opening because of the increased collimator scatter that was added to the primary beam [7]. It was seldom possible to measure the output factors for all the field sizes with decimal increments. For manual treatment time calculation in telecobalt units and linear accelerators, the in-water output was measured from 5 × 5 cm square field size to 35 × 35 cm and 5 × 5 cm to 40 × 40 cm, respectively, with equal interval of 5 cm^2. The output factors for any equivalent square field or square field could be calculated by applying linear interpolation method from the measured output values.

For computerized treatment time calculation by treatment planning system (TPS) the field size intervals of output factor measurements were recommended by vendors. The accuracy of dose calculation differs with available algorithm; however, the accurate measurement of basic beam data determines the accuracy of dose calculation by the TPS. In general, the output factors were measured at reference depth from 4 × 4 cm^2 field size to 40 × 40 cm^2 with the combination of square and rectangular fields. The intermediate output factor values were obtained by applying the linear interpolation method.

The requirement of interpolation arises when there is a situation of obtaining intermediate values from the available discrete data [x_0, x_1, x_2, x_3, x_4 ... x_{n+1}]. The process of obtaining a simple function f(x) $=a_0x+a_1x^2+a_2x^3+a_3x^3...$ $+a_nx^n$ that passes through all these discrete points is called an interpolation method.

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The intermediate values can be estimated from the obtained polynomial. Even though several methods of interpolation are available, the most suitable interpolation formulae were propounded by Newton and Lagrange [8]. At present, the measured output factors of discrete values were used for obtaining simultaneous linear equations, which is called direct method. However, the measured output factors against the field sizes follow higher degree polynomials.

Considering the output factor variation with field size, as a continuous function, it was worthwhile to use polynomial-based interpolation to calculate the intermediate values. Since output factor is a continuous function, the accuracy of linear interpolation method depends on the smallest interval taken between the data points, which demand more measurements with small interval. Applying higher degree polynomial interpolation may reduce the need of having small intervals between the data points. In this study, an attempt was made to investigate the application of higher degree polynomial interpolation methods (Newton backward and Lagrange methods) in calculating the output factors. The calculated values of output factors from all the three methods were compared with the measured values of intermediate output factors.

Materials and Methods

Ionization measurements were carried out at linear accelerator (Varian Medical Systems, Palo Alto, USA, Model:2100CD) for 6 MV X-rays at the depths of 5, 10, and 15 cm by source to surface distance (SSD) method using Farmer type ionisation chamber, 0.60 cm^3 (PTW Freiburg, S. No: 007023). The SSD of 100 cm is maintained for all the measurement depths. The ionisation chamber was placed at the above-mentioned depths in the radiation field analyzer filled with water. The output factor measurements were made for square fields from 4 × 4 cm^2 to 40 × 40 cm^2 field sizes with 0.5 cm^2 interval for the above-mentioned depths. The measured values of output factors of square fields 4 × 4 cm^2 to 40 × 40 cm^2 with 4 cm interval were taken for generating output factor table to estimate the in-between values (Figure 1, Table1).

The in-between values for all the square fields with 0.5 cm^2 interval were calculated by using traditional linear or direct interpolation method and the values were tabulated. In this study, a new attempt was made to estimate the intermediate values of the output factors by applying Lagrange and Newton backward interpolation methods and their accuracy levels were compared with the measured value.

Linear interpolation method

Linear interpolation is the simplest form of interpolation that connects two data points with a straight line [9]. It can also be called as the straight-line curve fit between two data points. This type of interpolation was used in filling the unmeasured intermediate values in the output factor table. From the discrete values of output factor values, namely x_0, x_1, x_2, x_3, x_4.... x_{n+1}, intermediate output factor values were found using the below formula.

$$f_1(x) = f(x_0) + \frac{f(x_1)-f(x_0)}{x_1-x_0}(x-x_0) \qquad (1)$$

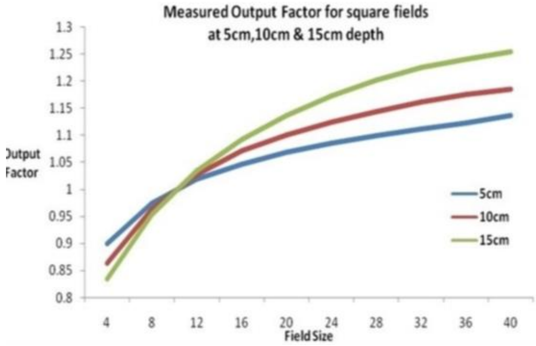

Figure 1.

Newton backward interpolation method

Newton backward interpolation method is applicable where equal interval of data sets is available. For the given set of (n+1) values such as (x_0, y_0), (x_1, y_1), (x_2, y_2) (x_3, y_3).... (x_n, y_n), a polynomial of n^{th} degree was obtained as

$$y_n(x) = y_n + p\Delta y_n + \frac{p(p+1)\ \Delta^2 y_n}{2!} + \frac{p(p+1)(p+2)\ \Delta^3 y_n}{3!} \ldots + \frac{p(p+1)\ldots\ldots(p+n-1)\ \Delta^n y_n}{n!} \qquad (2)$$

whereP= $(x-x_n)/h$, x- independent variable's (field size) output factor to be estimated, x_n-n^{th} is value of independent variable (field size), and h is the interval difference in data set. The values of Δy_n, $\Delta^2 y_n$, $\Delta^3 y_n$ $\Delta^4 y_n$ $\Delta^5 y_n$..... $\Delta^9 y_n$ were estimated from the difference table mentioned below. In our study, we estimated the intermediate values of $y_n(x)$ by programming the above-mentioned formulae in the MS excel software pack. The difference tables were formed as shown below.

Lagrange interpolation method

The Lagrange method is a popular method for polynomial-based interpolation [10]. It can be applied in both situations (equidistant and not equidistant) of the independent discrete values. For the given set of values, such as (x_0, y_0), (x_1, y_1), (x_2, y_2) (x_3, y_3).... (x_n, y_n), a polynomial of n^{th} degree was obtained by using the below formula.

$$\sum_{i=0}^{n}\left(\prod_{j=0,\ j\neq i}^{n}\frac{x-x_j}{x_i-x_j}\right)f_i \qquad (3)$$

The above-mentioned formula was programmed in MS excel software and the intermediate output factor values were estimated.

Table 1. Measured output factors at 5 cm depth taken for interpolation with interval of 4

Depth (cm)	Field size	4	8	12	16	20	24	28	32	36	40
5	Output factor	0.899	0.975	1.019	1.047	1.068	1.085	1.099	1.112	1.122	1.136
10	Output factor	0.863	0.963	1.027	1.071	1.100	1.124	1.144	1.162	1.174	1.185
15	Output factor	0.835	0.954	1.034	1.092	1.136	1.173	1.203	1.225	1.241	1.255

Results

The square field size output factor values mentioned in Table 1 were taken for estimating the intermediate output factors. The calculated output factors by Newton backward, Lagrange, and linear interpolation methods were tabulated in tables 2, 3, and 4 for 5 cm, 10 cm, and 15 cm, respectively. The percentage of variation of output factors from the measured values for all the three methods are mentioned in tables 2, 3, and 4.

Comparison in the field size range of 4 × 4 cm² to 10 × 10 cm²

Comparison of the measured output factors and calculated output factors at 5, 10, and 15 cm depths is shown in figures 2 a, b, and c for the field sizes from 4 × 4 cm² to 10 × 10 cm². It was evident that for field sizes ranging from 4 × 4 cm² to 10 × 10 cm². The Lagrange and Newton backward methods had good agreement with the measured value. The linear method underestimates the value for this range of field sizes irrespective of depth.

The comparison of percentage deviation for all the three methods in the field size ranges from 4 × 4 cm² to 10 × 10 cm² is shown in figures 2 d, e, and f for all the depths. The Lagrange and Newton backward methods had the maximum deviations of -0.10%, 0.5%, and 0.667% and average deviations of -0.03%, 0.15%, and 0.16% at the depths of 5, 10, and 15 cm, respectively; the linear method had the maximum deviations of -0.71%, -0.65%, and -0.809 and average deviations of -0.38%, -0.38%, -0.43%. It can be noted that the linear method underestimates the O.F at all the depths in this field size region.

While estimating the intermediate values, traditional linear interpolation method requires small intervals to reduce deviation from the measured value. Implementing polynomial-based interpolation for small field size regions can eliminate the need for using small intervals and increasing accuracy. The output factor function undergoes much variation in this small field size region as it follows higher degree polynomials. Accordingly, using linear interpolation at this range of field size should be reviewed and polynomial-based interpolation methods be adopted for finding the intermediate values.

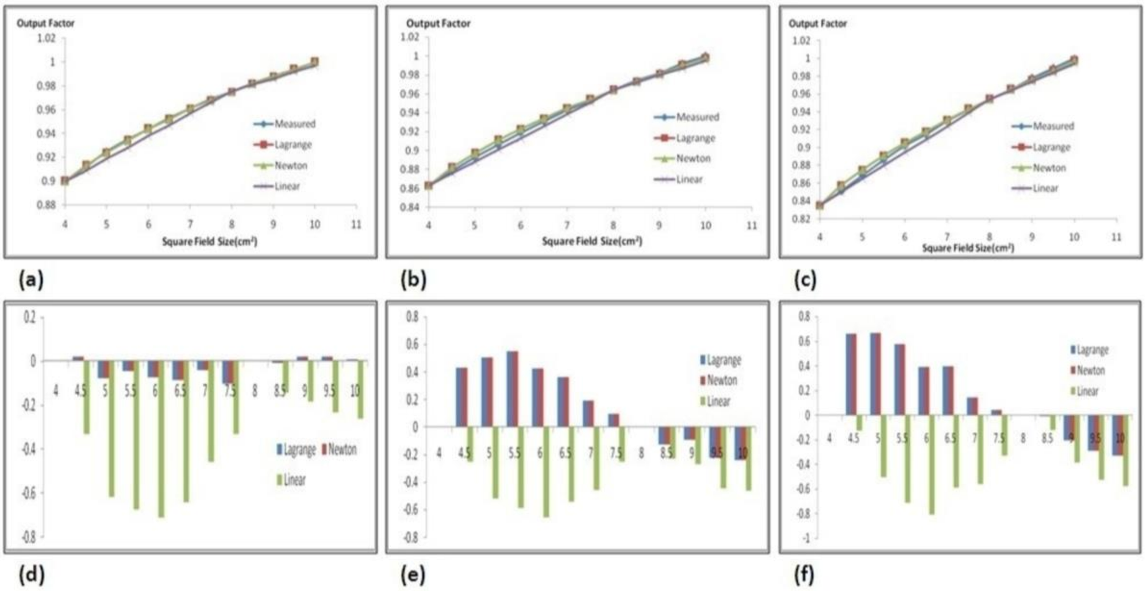

Figure 2.

Table 2. Analysis of calculated output factors of square fields for Newton Backward method, Lagrange method & linear method with the measured output factor at the depth of 5cm

		Calculated Output factor by interpolation methods			% of variation from measured Output Factors		
Field Size	Measured Output Factor	Newton Backward Method	Lagrange Method	Linear Method	Newton Backward Method	Lagrange Method	Linear
4	0.9	0.9	0.9	0.9	0	0	0
4.5	0.912	0.913	0.913	0.909	0.022	0.022	-0.329
5	0.925	0.924	0.924	0.919	-0.076	-0.076	-0.617
5.5	0.935	0.934	0.934	0.928	-0.043	-0.043	-0.674
6	0.944	0.944	0.944	0.938	-0.074	-0.074	-0.71
6.5	0.953	0.952	0.952	0.947	-0.084	-0.084	-0.64
7	0.961	0.961	0.961	0.957	-0.042	-0.042	-0.458
7.5	0.969	0.968	0.968	0.966	-0.103	-0.103	-0.33
8	0.975	0.975	0.975	0.975	0	0	0
8.5	0.982	0.982	0.982	0.981	-0.01	-0.01	-0.143
9	0.988	0.988	0.988	0.986	0.02	0.02	-0.182
9.5	0.994	0.994	0.994	0.992	0.02	0.02	-0.231
10	1	1	1	0.997	0.01	0.01	-0.26
10.5	1.005	1.005	1.005	1.003	0	0	-0.249
11	1.01	1.01	1.01	1.009	0.04	0.04	-0.149
11.5	1.015	1.015	1.015	1.014	0.03	0.03	-0.079
12	1.02	1.02	1.02	1.02	0	0	0
12.5	1.023	1.024	1.024	1.023	0.039	0.039	-0.02
13	1.027	1.028	1.028	1.026	0.039	0.039	-0.068
13.5	1.031	1.031	1.031	1.03	0.049	0.049	-0.078
14	1.034	1.035	1.035	1.033	0.039	0.039	-0.097
14.5	1.038	1.038	1.038	1.037	0.048	0.048	-0.067
15	1.041	1.041	1.041	1.04	0.038	0.038	-0.058
15.5	1.044	1.044	1.044	1.044	0.019	0.019	-0.029
16	1.047	1.047	1.047	1.047	0	0	0
16.5	1.05	1.05	1.05	1.05	0.01	0.01	-0.01
17	1.053	1.053	1.053	1.052	-0.009	-0.009	-0.047
17.5	1.055	1.055	1.055	1.055	-0.009	-0.009	-0.047
18	1.058	1.058	1.058	1.058	-0.009	-0.009	-0.047
18.5	1.061	1.061	1.061	1.06	0	0	-0.028
19	1.063	1.063	1.063	1.063	0.028	0.028	0
19.5	1.066	1.066	1.066	1.065	0	0	-0.019
20	1.068	1.068	1.068	1.068	0	0	0

Table 2.

		Calculated Output factor by interpolation methods			% of variation from measured Output Factors		
Field Size	Measured Output Factor	Newton Backward Method	Lagrange Method	Linear Method	Newton Backward Method	Lagrange Method	Linear
20.5	1.071	1.07	1.07	1.07	-0.065	-0.065	-0.084
21	1.073	1.073	1.073	1.072	-0.019	-0.019	-0.047
21.5	1.075	1.075	1.075	1.075	0.009	0.009	-0.028
22	1.077	1.077	1.077	1.077	0.009	0.009	-0.037
22.5	1.08	1.08	1.08	1.079	-0.019	-0.019	-0.065
23	1.082	1.082	1.082	1.081	-0.009	-0.009	-0.046
23.5	1.084	1.084	1.084	1.083	0	0	-0.018
24	1.086	1.086	1.086	1.086	0	0	0
24.5	1.088	1.088	1.088	1.087	0	0	-0.018
25	1.089	1.089	1.089	1.089	0.028	0.028	0
25.5	1.091	1.091	1.091	1.091	0.009	0.009	-0.027
26	1.093	1.093	1.093	1.093	0.009	0.009	-0.018
26.5	1.094	1.095	1.095	1.094	0.018	0.018	-0.009
27	1.096	1.096	1.096	1.096	0.009	0.009	-0.009
27.5	1.098	1.098	1.098	1.098	0	0	-0.009
28	1.099	1.099	1.099	1.099	0	0	0
28.5	1.101	1.101	1.101	1.101	0	0	0
29	1.103	1.102	1.102	1.103	-0.009	-0.009	0
29.5	1.104	1.104	1.104	1.104	0	0	0.009
30	1.106	1.106	1.106	1.106	-0.009	-0.009	0
30.5	1.107	1.107	1.107	1.107	-0.009	-0.009	0
31	1.109	1.109	1.109	1.109	0	0	0.009
31.5	1.111	1.111	1.111	1.111	0	0	0
32	1.112	1.112	1.112	1.112	0	0	0
32.5	1.114	1.114	1.114	1.113	0	0	-0.027
33	1.115	1.115	1.115	1.115	0.027	0.027	-0.027
33.5	1.116	1.117	1.117	1.116	0.045	0.045	-0.027
34	1.118	1.118	1.118	1.117	0.063	0.063	-0.018
34.5	1.119	1.12	1.12	1.119	0.072	0.072	-0.018
35	1.12	1.121	1.121	1.12	0.054	0.054	-0.018
35.5	1.121	1.122	1.122	1.121	0.045	0.045	0
36	1.122	1.122	1.122	1.122	0	0	0
36.5	1.124	1.123	1.123	1.124	-0.062	-0.062	-0.018
37	1.125	1.124	1.124	1.125	-0.124	-0.124	-0.027
37.5	1.126	1.124	1.124	1.126	-0.187	-0.187	-0.036
38	1.127	1.124	1.124	1.127	-0.24	-0.24	-0.027
38.5	1.128	1.125	1.125	1.128	-0.275	-0.275	-0.018
39	1.129	1.126	1.126	1.129	-0.266	-0.266	-0.009
39.5	1.13	1.128	1.128	1.13	-0.186	-0.186	-0.009
40	1.131	1.131	1.131	1.131	0	0	0

Comparison of the field size range of 10 × 10 cm² to 20 × 20 cm²

For field sizes from 10 × 10 cm² to 20 × 20 cm², the comparison was made in figures 3 a, b, and c. The Lagrange and Newton backward methods had the maximum deviations of 0.049%, -0.24%, and -0.327% and average deviations of -0.01%, 0.02%, and -0.051% (figures 3 d, e, and f) at the depths of 5, 10, and 15 cm, respectively. Similarly, linear method had the maximum deviations of -0.24%, -0.46%, and -0.56% and average deviations of -0.05%, -0.088%, and -0.165%. It was evident that the Newton backward and Lagrange methods have lower average deviation than the conventional linear interpolation method.

Table 3. Analysis of calculated output factors of square fields for Newton Backward method, Lagrange method & linear method with the measured output factor at the depth of 10cm

		Calculated Output factor by interpolation methods			% of variation from measured Output Factors		
Field Size	Measured Output Factor	Newton Backward Method	Lagrange Method	Linear Method	Newton Backward Method	Lagrange Method	Linear
4	0.863	0.863	0.863	0.863	0	0	0
4.5	0.878	0.882	0.882	0.876	0.433	0.433	-0.251
5	0.893	0.897	0.897	0.888	0.504	0.504	-0.515
5.5	0.906	0.911	0.911	0.901	0.552	0.552	-0.585
6	0.919	0.923	0.923	0.913	0.424	0.424	-0.653
6.5	0.931	0.934	0.934	0.926	0.365	0.365	-0.537
7	0.943	0.945	0.945	0.939	0.191	0.191	-0.456
7.5	0.953	0.954	0.954	0.951	0.094	0.094	-0.252
8	0.964	0.964	0.964	0.964	0	0	0
8.5	0.974	0.973	0.973	0.972	-0.123	-0.123	-0.226
9	0.982	0.981	0.981	0.98	-0.092	-0.092	-0.265
9.5	0.992	0.99	0.99	0.987	-0.222	-0.222	-0.444
10	1	0.998	0.998	0.995	-0.24	-0.24	-0.46
10.5	1.007	1.006	1.006	1.003	-0.139	-0.139	-0.358
11	1.014	1.013	1.013	1.011	-0.069	-0.069	-0.247
11.5	1.02	1.02	1.02	1.019	-0.01	-0.01	-0.108
12	1.027	1.027	1.027	1.027	0	0	0
12.5	1.033	1.034	1.034	1.033	0.048	0.048	-0.058
13	1.04	1.04	1.04	1.038	0	0	-0.173
13.5	1.047	1.046	1.046	1.044	-0.076	-0.076	-0.287
14	1.052	1.051	1.051	1.049	-0.01	-0.01	-0.228
14.5	1.057	1.057	1.057	1.055	-0.038	-0.038	-0.246
15	1.061	1.062	1.062	1.06	0.085	0.085	-0.085
15.5	1.065	1.067	1.067	1.066	0.131	0.131	0.038
16	1.071	1.071	1.071	1.071	0	0	0
16.5	1.074	1.075	1.075	1.075	0.112	0.112	0.056
17	1.078	1.079	1.079	1.079	0.093	0.093	0.009
17.5	1.083	1.083	1.083	1.082	0.065	0.065	-0.028
18	1.086	1.087	1.087	1.086	0.092	0.092	-0.009
18.5	1.09	1.091	1.091	1.09	0.083	0.083	-0.009
19	1.094	1.094	1.094	1.093	0.046	0.046	-0.018
19.5	1.097	1.098	1.098	1.097	0.027	0.027	-0.009
20	1.101	1.101	1.101	1.101	0	0	0

Table 3.

		Calculated Output factor by interpolation methods			% of variation from measured Output Factors		
Field Size	Measured Output Factor	Newton Backward Method	Lagrange Method	Linear Method	Newton Backward Method	Lagrange Method	Linear
20.5	1.104	1.104	1.104	1.104	-0.027	-0.027	-0.045
21	1.108	1.107	1.107	1.107	-0.045	-0.045	-0.081
21.5	1.11	1.11	1.11	1.11	0.009	0.009	-0.036
22	1.114	1.113	1.113	1.113	-0.018	-0.018	-0.072
22.5	1.116	1.116	1.116	1.116	-0.018	-0.018	-0.063
23	1.119	1.119	1.119	1.119	0	0	-0.036
23.5	1.122	1.122	1.122	1.122	0	0	-0.027
24	1.125	1.125	1.125	1.125	0	0	0
24.5	1.127	1.127	1.127	1.127	0	0	-0.027
25	1.13	1.13	1.13	1.13	0.027	0.027	0
25.5	1.132	1.132	1.132	1.132	0	0	-0.044
26	1.134	1.135	1.135	1.134	0.035	0.035	0
26.5	1.137	1.137	1.137	1.137	0.035	0.035	0
27	1.139	1.14	1.14	1.139	0.035	0.035	0
27.5	1.142	1.142	1.142	1.142	0.018	0.018	0.009
28	1.144	1.144	1.144	1.144	0	0	0
28.5	1.147	1.146	1.146	1.146	-0.026	-0.026	-0.026
29	1.149	1.149	1.149	1.149	-0.035	-0.035	-0.035
29.5	1.151	1.151	1.151	1.151	-0.052	-0.052	-0.061
30	1.154	1.153	1.153	1.153	-0.061	-0.061	-0.069
30.5	1.157	1.155	1.155	1.155	-0.121	-0.121	-0.13
31	1.158	1.157	1.157	1.157	-0.069	-0.069	-0.069
31.5	1.16	1.159	1.159	1.159	-0.017	-0.017	-0.026
32	1.162	1.162	1.162	1.162	0	0	0
32.5	1.164	1.164	1.164	1.163	0.009	0.009	-0.026
33	1.165	1.166	1.166	1.165	0.026	0.026	-0.043
33.5	1.167	1.168	1.168	1.167	0.051	0.051	-0.034
34	1.169	1.169	1.169	1.168	0.068	0.068	-0.026
34.5	1.17	1.171	1.171	1.17	0.077	0.077	-0.026
35	1.172	1.173	1.173	1.172	0.068	0.068	-0.017
35.5	1.173	1.174	1.174	1.173	0.051	0.051	-0.009
36	1.175	1.175	1.175	1.175	0	0	0
36.5	1.177	1.176	1.176	1.176	-0.076	-0.076	-0.034
37	1.178	1.176	1.176	1.177	-0.161	-0.161	-0.068
37.5	1.18	1.177	1.177	1.179	-0.229	-0.229	-0.068
38	1.181	1.177	1.177	1.18	-0.263	-0.263	-0.042
38.5	1.182	1.178	1.178	1.181	-0.33	-0.33	-0.068
39	1.183	1.179	1.179	1.183	-0.304	-0.304	-0.042
39.5	1.185	1.182	1.182	1.184	-0.253	-0.253	-0.059
40	1.185	1.185	1.185	1.185	0	0	0

Table 4. Analysis of calculated output factors of square fields for Newton Backward, Lagrange & linear methods with the measured output factor at the depth of 15cm

		Calculated Output factor by interpolation methods			% of variation from measured Output Factors		
Field Size	Measured Output Factor	Newton Backward Method	Lagrange Method	Linear Method	Newton Backward Method	Lagrange Method	Linear
4	0.835	0.835	0.835	0.835	0	0	0
4.5	0.851	0.857	0.857	0.85	0.658	0.658	-0.129
5	0.869	0.875	0.875	0.865	0.667	0.667	-0.506
5.5	0.886	0.891	0.891	0.88	0.576	0.576	-0.711
6	0.902	0.905	0.905	0.895	0.388	0.388	-0.809
6.5	0.915	0.918	0.918	0.909	0.394	0.394	-0.59
7	0.93	0.931	0.931	0.924	0.14	0.14	-0.559
7.5	0.942	0.943	0.943	0.939	0.042	0.042	-0.329
8	0.954	0.954	0.954	0.954	0	0	0
8.5	0.965	0.965	0.965	0.964	-0.01	-0.01	-0.124
9	0.978	0.976	0.976	0.974	-0.205	-0.205	-0.389
9.5	0.989	0.986	0.986	0.984	-0.293	-0.293	-0.526
10	1	0.997	0.997	0.994	-0.33	-0.33	-0.58
10.5	1.01	1.007	1.007	1.004	-0.327	-0.327	-0.564
11	1.017	1.016	1.016	1.014	-0.098	-0.098	-0.295
11.5	1.027	1.025	1.025	1.024	-0.107	-0.107	-0.214
12	1.034	1.034	1.034	1.034	0	0	0
12.5	1.043	1.043	1.043	1.042	0	0	-0.125
13	1.051	1.051	1.051	1.049	0.029	0.029	-0.171
13.5	1.058	1.059	1.059	1.056	0.019	0.019	-0.227
14	1.066	1.066	1.066	1.063	0.028	0.028	-0.225
14.5	1.073	1.073	1.073	1.07	0.019	0.019	-0.205
15	1.08	1.08	1.08	1.078	-0.019	-0.019	-0.194
15.5	1.086	1.086	1.086	1.085	-0.037	-0.037	-0.138
16	1.092	1.092	1.092	1.092	0	0	0
16.5	1.098	1.098	1.098	1.097	-0.027	-0.027	-0.073
17	1.105	1.104	1.104	1.103	-0.127	-0.127	-0.19
17.5	1.11	1.109	1.109	1.108	-0.099	-0.099	-0.18
18	1.115	1.115	1.115	1.114	-0.036	-0.036	-0.108
18.5	1.121	1.12	1.12	1.119	-0.071	-0.071	-0.143
19	1.126	1.125	1.125	1.125	-0.071	-0.071	-0.124
19.5	1.132	1.13	1.13	1.13	-0.097	-0.097	-0.124
20	1.136	1.136	1.136	1.136	0	0	0

Table 4.

		Calculated Output factor by interpolation methods			% of variation from measured Output Factors		
Field Size	Measured Output Factor	Newton Backward Method	Lagrange Method	Linear Method	Newton Backward Method	Lagrange Method	Linear
20.5	1.142	1.141	1.141	1.14	-0.088	-0.088	-0.105
21	1.146	1.146	1.146	1.145	-0.017	-0.017	-0.061
21.5	1.151	1.15	1.15	1.15	-0.009	-0.009	-0.061
22	1.154	1.155	1.155	1.155	0.078	0.078	0.009
22.5	1.16	1.16	1.16	1.159	0.043	0.043	-0.017
23	1.164	1.165	1.165	1.164	0.052	0.052	0
23.5	1.168	1.169	1.169	1.169	0.068	0.068	0.043
24	1.174	1.174	1.174	1.174	0	0	0
24.5	1.178	1.178	1.178	1.177	-0.008	-0.008	-0.051
25	1.183	1.182	1.182	1.181	-0.059	-0.059	-0.135
25.5	1.187	1.186	1.186	1.185	-0.11	-0.11	-0.202
26	1.19	1.19	1.19	1.188	-0.042	-0.042	-0.143
26.5	1.193	1.193	1.193	1.192	-0.017	-0.017	-0.117
27	1.197	1.197	1.197	1.196	-0.042	-0.042	-0.117
27.5	1.199	1.2	1.2	1.199	0.033	0.033	-0.008
28	1.203	1.203	1.203	1.203	0	0	0
28.5	1.206	1.206	1.206	1.206	0.025	0.025	0
29	1.209	1.209	1.209	1.209	0.008	0.008	-0.025
29.5	1.212	1.212	1.212	1.211	-0.025	-0.025	-0.066
30	1.214	1.215	1.215	1.214	0.074	0.074	0.033
30.5	1.218	1.217	1.217	1.217	-0.049	-0.049	-0.09
31	1.221	1.22	1.22	1.22	-0.066	-0.066	-0.098
31.5	1.223	1.223	1.223	1.223	-0.033	-0.033	-0.049
32	1.225	1.225	1.225	1.225	0	0	0
32.5	1.227	1.228	1.228	1.227	0.033	0.033	-0.016
33	1.23	1.23	1.23	1.229	0.049	0.049	-0.033
33.5	1.232	1.233	1.233	1.231	0.057	0.057	-0.057
34	1.234	1.235	1.235	1.233	0.057	0.057	-0.065
34.5	1.236	1.237	1.237	1.235	0.032	0.032	-0.089
35	1.238	1.238	1.238	1.237	0.032	0.032	-0.073
35.5	1.24	1.24	1.24	1.239	0.008	0.008	-0.048
36	1.241	1.241	1.241	1.241	0	0	0
36.5	1.243	1.242	1.242	1.243	-0.064	-0.064	0.008
37	1.244	1.242	1.242	1.244	-0.113	-0.113	0.04
37.5	1.245	1.243	1.243	1.246	-0.177	-0.177	0.08
38	1.247	1.244	1.244	1.248	-0.281	-0.281	0.064
38.5	1.249	1.245	1.245	1.25	-0.344	-0.344	0.064
39	1.251	1.246	1.246	1.251	-0.336	-0.336	0.064
39.5	1.253	1.25	1.25	1.253	-0.24	-0.24	0.048
40	1.255	1.255	1.255	1.255	0	0	0

It was evident that all the interpolation methods had similar percentage of deviation from the measured value. The output factor function starts approaching linearity in this field region, and thus, the difference was insignificant. However, the average deviation of linear interpolation was still higher when compared to Lagrange and Newton backward methods. It is recommended to use polynomial-based interpolation in these field regions, as well.

Comparison of the field size range of 20 × 20 cm² to 30 × 30 cm²

The comparison of field sizes from 20 × 20 cm² to 30 × 30 cm² (figures 4 a, b, and c) for the Lagrange and Newton backward methods had the maximum deviations of -0.065%, -0.121%, and -0.11% and average deviations of 0.00%, -0.01%, and -0.001% (figures 4 d, e, and f) at the depths of 5, 10, and 15 cm, respectively.

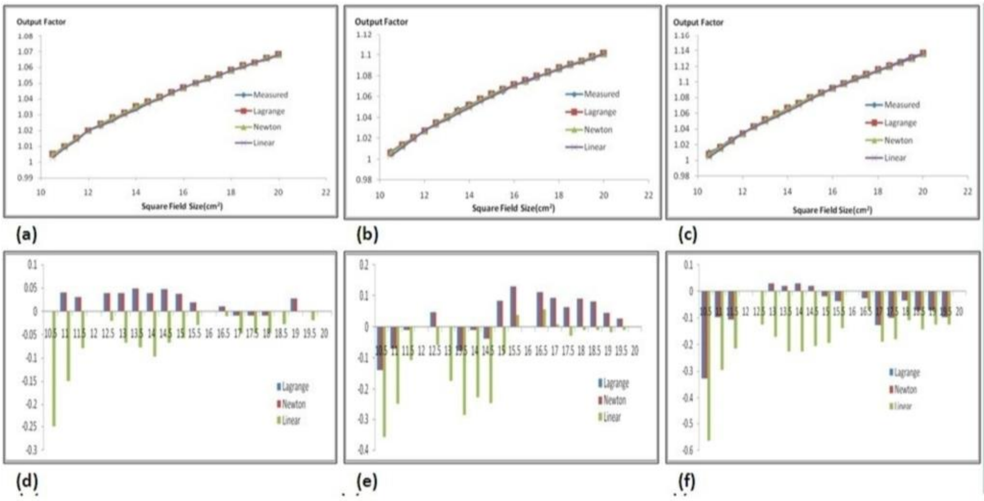

Figure 3.

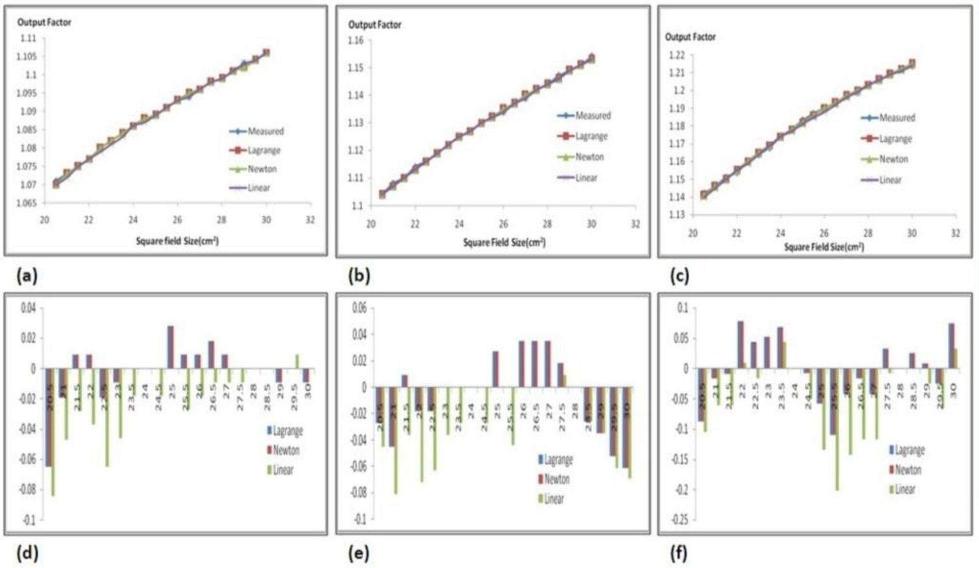

Figure 4.

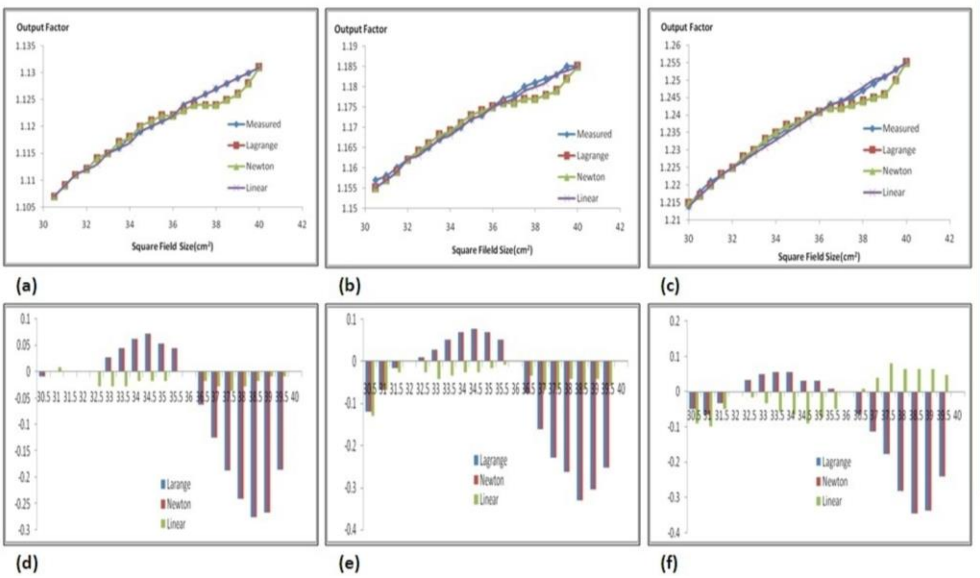

Figure 5.

Table 5. Statistical Analysis for interpolation methods at 5cm depth

	5cm			10cm			15cm		
	Lagrange Method	Newton Backward Method	Linear Method	Lagrange Method	Newton Backward Method	Linear Method	Lagrange Method	Newton Backward Method	Linear Method
Mean Deviation from the Measured O.F	0.01493 ± 0.0696	-0.01493 ± 0.0696	-0.0868 ± 0.1647	0.0100 ± 0.158	0.0100 ± 0.158	-0.106 ± 0.161	-0.0063 ± 0.1839	-0.0063± 0.1839	-0.1345± 0.1934
P Value	0.0963	0.0963	<0.005	0.999	0.999	<0.005	0.485	0.485	<0.005

Similarly linear method had the maximum deviations of -0.08%, -0.13%, and -0.20% and average deviations -0.016%, -0.034%, and -0.05%. The observation of these figures shows non-uniform deviations in terms of positive and negative deviations from the measured values.

Even though the comparison diagram demonstrates that all the interpolation methods were in good agreement with the measured value. The calculation of percentage of deviation clearly indicates that Newton backward and Lagrange methods have 0% deviation from the measured value. It was emphasized to use the proposed methods in this field size region.

Comparison in the field size range of 30 × 30 cm² to 40 × 40 cm²

For field sizes from 30 × 30 cm² to 40 × 40 cm² (figures 5.a, b, and c), the Lagrange and Newton backward methods had the maximum deviations of -0.275%, -0.33%, and 0.34% and average deviations of -0.05%, -0.071%, and -0.071% at the depths of 5, 10, and 15 cm, respectively (figures d, e, and f). Likewise, linear method had the maximum deviations of -0.086%, -0.13%, and -0.09% and average deviations of -0.017%, -0.034%, and -0.012%. It can be noted from the figures that the Lagrange and Newton backward methods underestimate the output factors from the field size of 36.5 cm² to 40 × 40 cm².

Statistical evaluation of the three methods with the measured values

Paired t-test was applied to check the significance of these methods. The calculated output factors by the Lagrange and Newton backward methods at 5cm,10cm & 15cm were insignificantly different (P=0.0963, 0.999, and 0.485 for 5cm,10cm & 15cm depths Table 5). The calculated O.F by linear interpolation method was significantly different from the measured value (P<0.005 at all the three depths).

Discussion

Field size range of 4 × 4 cm² to 10 × 10 cm²

The output factor function undergoes much variation in this small field size region as it follows higher degree polynomials. Therefore, the curves of estimated O.F by linear methods show higher deviation from the measured values. While estimating the intermediate values, traditional linear interpolation method requires small intervals to reduce deviation from the measured value. Implementing polynomial-based interpolation for small field size regions can eliminate the need for using small intervals and increasing accuracy. Accordingly, use of linear interpolation for this range of field size should be reviewed and polynomial-based interpolation methods be adopted for finding the intermediate values.

Field size range of 10 × 10 cm² to 20 × 20 cm²

The output factor function starts approaching linearity in this field region, and hence, the difference was insignificant. It was evident that all the interpolation methods had similar percentage of deviation from the measured value. However, the average deviation of linear interpolation was still higher when compared to Lagrange and Newton backward methods. It was recommended to use polynomial-based interpolation in these field regions, as well.

Field size range of 20 × 20 cm² to 30 × 30 cm²

The output factor function still follows linearity as the curves of linear interpolation have good agreement with the measured values. Even though the comparison diagram provides an understanding that all the interpolation methods were in good agreement with the measured value, the calculation of percentage of deviation clearly indicates that Newton backward and Lagrange methods have 0% deviation from the measured value. It was emphasized to use the proposed methods in this field size region.

Field size range of 30 × 30 cm² to 40 × 40 cm²

The Lagrange and Newton backward methods were not in good agreement with the measured values in these field size regions, particularly from 36.5 cm² to 40 cm². This may be due to computing greater number of data values since error increases with higher number of values. It was suggested not to use polynomial-based interpolations in these field size regions and instead use traditional linear interpolation method.

Estimation of unmeasured or missing values requires interpolation; linear interpolation is widely used as the method of determining this missing values. Various methods of interpolation are available and their applicability in estimating the unmeasured output factor was studied in this paper. The linearity of output factor with field size was not found in all the field size ranges. The O.F undergoes rapid changes in the field size region of 4 × 4 cm^2 to 10 × 10 cm^2. Missing values can be found by applying higher order interpolation methods if interval measurements are kept higher than conventional intervals. In our study, we found that application of higher order interpolation techniques yields close results to the measured values, while maintaining higher and reasonable field size intervals. However, the traditional use of linear interpolation does not give much significant deviation from the measured value, which is due to having close intervals when measuring the O.F values. To increase accuracy, the field size intervals between two measurements should be kept to a minimum in case of linear interpolation.

Conclusion

Output factor is an important parameter for delivering the prescribed dose to the tumor. High accuracy was expected while estimating the intermediate and unmeasured output factors. Applying interpolation is highly essential for estimating the unmeasured values. We concluded that the output factors estimated through Lagrange and Newton backward methods have better agreement with the measured value than the routinely used direct/linear interpolation method. Conceptually, linear interpolation can be applied for linear functions, and output factor follows higher order polynomials. Therefore, missing values in output factors can be found with higher accuracy by using Lagrange and Newton backward methods. In traditional approach, while using the linear interpolation, accuracy is maintained by keeping small or fine intervals in field size. The complex Lagrange interpolation & Newton backward method can be programmed in the MS excel software to minimize the computation time.

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